

Optimal Spectrum Sensing Threshold for Unequal Priors Case

Farrukh Javed¹, Imran Shafi¹ and Asad Mahmood²

¹Center for Advanced Studies in Engineering Islamabad, Pakistan,

²School of Computer Science, University of Witwatersrand,
Johannesburg, South Africa.

farrukh.javed@case.edu.pk, imranshafi@iqraisb.edu.pk, asad.mahmood@wits.ac.za

Abstract: Classical spectrum sensing techniques utilize maximum likelihood (ML) detection for identification of spectrum holes. The approach is sub-optimal for the case of un-equal priors where the probabilities of channel occupation and vacancy are not the same. Such situations are bound to occur in most commercial bands such as GSM etc and hence are of more interest. The loss in performance has been disregarded as negligible in most of the work done on spectrum sensing techniques. This paper quantifies the effects of changing priors on classical energy detection and infers that the loss in spectrum sensing performance is not negligible. The deterioration is especially considerable at low SNR values and at low probabilities of channel occupation. This paper aims at deriving an optimum threshold for achieving minimum probability of error for unequal prior case. Detection based on the proposed threshold out-performs classical detectors under the assumption that priors are known at the receiver.

Keywords: Cognitive Radios, Spectrum Sensing, ML Detection, Detection Threshold.

1. Introduction

Rapid expansion in communications has caused an apparent spectrum scarcity whereby, the available spectrum has already been allocated to potential users by various governing agencies. Analysis has revealed that this apparent scarcity is attributable to the inefficient fixed spectrum allocation techniques. This implies that although the spectrum has been licensed to various users but a major portion of this licensed spectrum remains under-utilized. For example, Federal communication commission (FCC) places the spectrum usage in USA between the ranges 15% to 85% at all times [6]. This has opened a new avenue in research to explore more efficient but complex dynamic spectrum access techniques. Dynamic spectrum access envisions the use of licensed spectral bands by smart unlicensed cognitive users that can exploit any opportunities that may exist in the form of temporal or spatial holes. A spectrum hole is that part of the spectrum where the primary users' transmission strength falls below a certain regulated level termed as interference cap by FCC [6]. The smart nodes that form the secondary users are called cognitive radios [7]. A cognitive radio is an evolved software defined radio that in addition to reconfiguration capability also possesses the ability to analyze its surrounding radio environment. This allows the cognitive radio to decide how best to reconfigure itself in existing radio conditions. The capability of a cognitive radio to adapt to its surroundings greatly depends upon the amount and accuracy of information it can acquire about its radio environment. The process by which the cognitive radio

becomes aware of its surroundings is termed as spectrum sensing and is a key challenge for cognitive radios. The prime objective of spectrum sensing is to identify the presence or absence of a licensed (primary) user in a channel. Various spectrum sensing algorithms have been suggested each with its own strengths and weaknesses [1]-[3]. Most of these algorithms utilize some sort of probabilistic modeling to detect the primary user in the observed spectrum [1]. These decision models are generally based on Maximum Likelihood (ML) detection whereby the likelihood probabilities of the signal and noise form the basis of detection [4]. The section II of this paper describes classical ML detection. The section III indicates various performance metrics used in literature to judge the accuracy of the spectrum sensing algorithms [5], [8]. Section IV analyzes classical approaches to decide the threshold level used to discriminate the noise from the signal and their related performance metrics. We have re-derived these metrics in terms of SNR and buffer size to make the comparison more meaningful. An optimal minimum probability of error detector has been suggested in section V. Section VI gives the simulation based results.

2. ML Detection

The ML detection is a binary hypothesis test to confirm the presence or absence of the signal $s(n)$ in a buffer size of N_B samples. Assuming that the received signal $y(n)$ is being tested for the presence or absence of $s(n)$ in white Gaussian noise $z(n)$, the hypotheses would be

$$\begin{aligned} h_0 : y(n) &= z(n) \\ h_1 : y(n) &= s(n) + z(n). \end{aligned} \quad (1)$$

If $p(D|h_i)$ is the likelihood of receiving test statistic D for the hypothesis h_i , then its probability density function $p(D)$ would be the weighted sum of two conditional likelihood probabilities $p(D|h_i)$.

$$p(D) = p(D|h_0)P(h_0) + p(D|h_1)P(h_1) \quad (2)$$

As we are assuming different variances for the two likelihood distributions, $D = \sum_{n=1}^{N_B} |y(n)|^2$ would be a sufficient test statistics for confirming the hypothesis [14]. Assuming that the samples of $s(n)$ and $z(n)$ are independent and identically distributed Gaussian in

amplitude $s(n) \sim \mathcal{N}(\mu_s, \sigma_s)$, $z(n) \sim \mathcal{N}(\mu_z, \sigma_z)$ then D would be the sum of N_B squared Gaussian samples and would therefore be Chi - Square distributed with N_B degrees of freedom $\chi_{N_B}^2$. The Chi-Square distribution can be approximated as asymptotically Gaussian $p(D|h_i) \sim \mathcal{N}(\mu_{D_i}, \sigma_{D_i}^2)$ in accordance with central limit theorem [12] for a large buffer size N_B . The mean μ_{D_i} and variances $\sigma_{D_i}^2$ of the two likelihood distributions $p(D|h_i)$ can easily be deduced using the definition of variance and the central limiting theorem, however the proof is not given here because of lack of space.

$$\mu_{D_0} = N_B(\mu_z^2 + \sigma_z^2) \quad (3)$$

$$\mu_{D_1} = N_B[(\mu_z + \mu_s)^2 + \sigma_z^2 + \sigma_s^2] \quad (4)$$

$$\sigma_{D_0}^2 = 2N_B(2\mu_z^2\sigma_z^2 + \sigma_z^4) \quad (5)$$

$$\sigma_{D_1}^2 = 2N_B[2(\mu_z + \mu_s)^2(\sigma_z^2 + \sigma_s^2) + (\sigma_z^2 + \sigma_s^2)^2] \quad (6)$$

3. Performance Metrics

The various performance metrics used in literature [5] for comparing performance of the spectrum sensing algorithms are summarized below.

3.1 Probability of False Alarm (P_{fa})

The probability false alarm is a measure of missed opportunities. This is important from a secondary user perspective as it indicates the failure to identify presence of a spectrum hole, while actually it did exist.

$$\begin{aligned} P_{fa} &= P(h_1|h_0) \\ &= P(D > \lambda | h_0) \\ &= \int_{\lambda}^{\infty} p(D|h_0) dD \\ &= Q\left(\frac{\lambda - \mu_{D_0}}{\sigma_{D_0}}\right) \end{aligned} \quad (7)$$

where Q-Function is the tail probability of the standard normal distribution and is λ is the sensing threshold.

3.2 Probability of Missed Detection (P_{md})

The probability of missed detection is a measure of failure to identify an existing primary user in a channel.

$$\begin{aligned} P_{md} &= P(h_0|h_1) \\ &= P(D < \lambda | h_1) \\ &= \int_{-\infty}^{\lambda} p(D|h_1) dD \\ &= Q\left(-\frac{\lambda - \mu_{D_1}}{\sigma_{D_1}}\right) \end{aligned} \quad (8)$$

3.3 Receiver Operating Curve (ROC)

Receiver operating curve is another conventional method of summarizing the performance of a detector. It is simply a plot of P_D versus P_{fa} where $P_D = 1 - P_{md}$ is the probability of detection.

$$P_D = Q\left(\frac{\lambda - \mu_{D_1}}{\sigma_{D_1}}\right) \quad (9)$$

3.4 Probability of Error (P_e)

The probability of error is the overall error measure obtained by weighted sum of P_{md} and P_{fa} . The weights

are given according to the prior probabilities of presence of a hole or primary user in the channel.

$$\begin{aligned} P_e &= P(h_1|h_0)P(h_0) + P(h_0|h_1)P(h_1) \\ &= P(h_0)Q\left(\frac{\lambda - \mu_{D_0}}{\sigma_{D_0}}\right) + P(h_1)Q\left(-\frac{\lambda - \mu_{D_1}}{\sigma_{D_1}}\right) \end{aligned} \quad (10)$$

3.5 Sensing Error Floor (SEF)

The minimum detection error obtained by choosing a sensing threshold λ_{SEF} that may result in equal error probabilities of false detection and missed detection is known as sensing error floor (SEF).

$$\begin{aligned} SEF &\equiv P(h_1|h_0) = P(h_0|h_1) \\ SEF &\equiv Q\left(\frac{\lambda - \mu_{D_0}}{\sigma_{D_0}}\right) = Q\left(-\frac{\lambda - \mu_{D_1}}{\sigma_{D_1}}\right) \end{aligned} \quad (11)$$

4. Literature Review

The aim of hypothesis testing is to devise a sensing threshold λ that suitably divides the two generally overlapping distributions in $p(D)$ while optimizing a certain selected performance metrics. For a sensing threshold λ the ML detection model is

$$L_R(D) = \frac{p(D|h_1)}{p(D|h_0)} \stackrel{h_0}{\underset{h_1}{\leq}} T(\lambda) \quad (12)$$

where L_R is termed as the likelihood ratio of h_0 versus h_1 for different values of D . $T(\lambda)$ is the ratio of likelihood distributions at sensing threshold $D = \lambda$; $T(\lambda) = p(\lambda|h_1)/p(\lambda|h_0)$.

4.1 Neyman - Pearson Theorem

An application of Neyman - Pearson theorem [9], [14] – [16], aims at optimizing probability of detection P_D under the assumption that a certain fixed level of $P_{fa} = \alpha$ has been assumed as acceptable. The advantage of this approach is that the sensing threshold $\lambda_{P_{fa}}$ can be derived simply from the noise parameters without any knowledge of primary user transmission. This implies a non-parametric, blind spectrum sensing. The sensing threshold λ_{NP}, P_{md} and P_D in this case can be deduced as

$$\lambda_{NP} = \sigma_{D_0}Q^{-1}(\alpha) + \mu_{D_0} \quad (13)$$

$$\begin{aligned} P_{md} &= Q\left(-\frac{\lambda_{NP} - \mu_{D_1}}{\sigma_{D_1}}\right) \\ &= Q\left(-\frac{\sigma_{D_0}}{\sigma_{D_1}}Q^{-1}(\alpha) - \frac{\mu_{D_0} - \mu_{D_1}}{\sigma_{D_1}}\right) \end{aligned} \quad (14)$$

$$P_D = Q\left(\frac{\sigma_{D_0}}{\sigma_{D_1}}Q^{-1}(\alpha) + \frac{\mu_{D_0} - \mu_{D_1}}{\sigma_{D_1}}\right) \quad (15)$$

SEF in this case cannot be defined in this case as P_{fa} is fixed while the consolidated probability of error is

$$P_e = \alpha P(h_0) + P(h_1)Q\left(-\frac{\sigma_{D_0}}{\sigma_{D_1}}Q^{-1}(\alpha) - \frac{\mu_{D_1} - \mu_{D_0}}{\sigma_{D_1}}\right). \quad (16)$$

4.2 Sensing Threshold for Achieving SEF

A more accurate ML classification approach [4] suggests selection of a sensing threshold λ_{SEF} that results in achieving sensing error floor under the assumption that $P_{fa} = P_{md}$. Sensing threshold λ_{SEF} can be deduced in this case by equating Eqn. (7) and Eqn. (8).

$$\lambda_{SEF} = \frac{\mu_{D_1}\sigma_{D_0} + \mu_{D_0}\sigma_{D_1}}{\sigma_{D_0} + \sigma_{D_1}} \quad (17)$$

P_{fa}, P_{md}, P_e and SEF are all equal in this case and are according to Eqn. (11). P_D may be found from either noise or signal parameters.

$$P_D = Q\left(-\frac{\lambda_{SEF} - \mu_{D_0}}{\sigma_{D_0}}\right) = Q\left(\frac{\lambda_{SEF} - \mu_{D_1}}{\sigma_{D_1}}\right) \quad (18)$$

4.3 Minimum P_e Detector

The minimum P_e detector is a special case of a more general Bayesian detector which aims at optimizing the P_e assuming that the prior probabilities of hypotheses are known [14]. The likelihood ratio for minimum P_e detector is given as

$$T(\lambda_{P_e}) = \frac{P(h_0)}{P(h_1)} \quad (19)$$

and the detection model becomes

$$P(h_1)p(D|h_1) \underset{h_1}{\leq} P(h_0)p(D|h_0). \quad (20)$$

The sensing threshold λ_{P_e} can also be deduced in terms of parameters of likelihood distributions $p(D|h_i)$ by considering that $P(h_0)p(\lambda_{P_e}|h_0) = P(h_1)p(\lambda_{P_e}|h_1)$. This results into a quadratic equation that can be simplified to get the sensing threshold as

$$\lambda_{P_e} = \frac{\mu_{D_0} + \mu_{D_1}}{\sigma_{D_1}^2 - \sigma_{D_0}^2} + \left[\left(\frac{\mu_{D_0} + \mu_{D_1}}{\sigma_{D_1}^2 - \sigma_{D_0}^2} \right)^2 + \frac{\mu_{D_0}^2\sigma_{D_1}^2 + \mu_{D_1}^2\sigma_{D_0}^2 + \sigma_{D_1}^2\sigma_{D_0}^2 \ln(\beta\sqrt{2SNR} + 1)}{\sigma_{D_1}^2 - \sigma_{D_0}^2} \right]^{\frac{1}{2}} \quad (21)$$

where $\beta = P(h_0)/P(h_1)$ is the ratio of priors. The performance metrics in this case are given by Eqns. (7)–(11) with $\lambda = \lambda_{P_e}$, further simplification not being possible.

4.4 Comparison of the Sensing Thresholds

P_e is considered here as the most suitable performance metric for comparison. In order to make the comparison more meaningful the P_e can be reflected in terms of buffer size N_B and SNR . Under some reasonable assumptions, P_e for the three sensing thresholds can be deduced using Eqns. (3)–(6) as

$$P_e(\lambda_{NP}) = \alpha P(h_0) + P(h_1) Q\left(\frac{-1}{\sqrt{2SNR+1}} \left[Q^{-1}(\alpha) - \sqrt{\frac{N_B}{2} SNR} \right] \right) \quad (22)$$

$$P_e(\lambda_{SEF}) = Q\left(\frac{\sqrt{\frac{N_B}{2} SNR}}{1 + \sqrt{\frac{\gamma-1}{2} SNR^2 + 2SNR + 1}}\right) \quad (23)$$

where $\gamma = E[|s(n)|^4]/\{E[|s(n)|^2]\}^2$ is a measure of randomness of the signal [4] and varies between 1 to 2. For deducing $P_e(\lambda_{P_e})$ in terms of SNR and N_B we first convert Eqn. (21) in terms of SNR and N_B .

$$\lambda_{P_e} = \frac{1}{\sigma_z^2} \left(1 + \frac{2}{SNR} \right) + \left[\frac{1}{\sigma_z^2} \left(1 + \frac{2}{SNR} \right)^2 + N_B \sigma_z^4 \left\{ \frac{SNR}{2} + \left(2 + \frac{1}{SNR} \right) \ln(\beta\sqrt{2SNR} + 1) \right\} \right]^{1/2} \quad (24)$$

$$P_e(\lambda_{P_e}) = P(h_0) Q\left(\frac{\lambda_{P_e}}{\sigma_z^2 \sqrt{2N_B}} - \sqrt{\frac{N_B}{2}}\right) + P(h_{-1})$$

$$Q\left(\frac{-1}{\sqrt{2SNR+1}} \left[\frac{\lambda_{P_e}}{\sigma_z^2 \sqrt{2N_B}} - \sqrt{\frac{N_B}{2}} (SNR + 1) \right] \right) \quad (25)$$

The comparison of the analytical results for P_e for the three classical sensing thresholds at $P(h_0) = 0.7$ has been displayed in Figure 1. The figure elaborates that for higher SNR and N_B the P_e reduces to zero for all three cases. For low SNR and N_B the P_e is maximum for λ_{NP} and λ_{SEF} .

5. Derivation of Optimal Threshold for Unequal Priors Case

As implied by Eqn. (2) the received test statistic is the sum of two joint distributions $p(D) = p(D, h_0) + p(D, h_1)$. The minimum P_e detector described in Eqn. (20) attempts to bifurcate the two distributions by taking their point of intersection as the sensing threshold λ_{P_e} . However as the two distributions acquire distinct parameters; the point of intersection is not an optimal threshold anymore. We propose an optimal sensing threshold for the Bayesian case (with known priors) which aims at minimizing the joint detection error. Assuming a sensing threshold λ_{opt} , the suggested Maximum A Posterior Probability (MAP) detection model is:

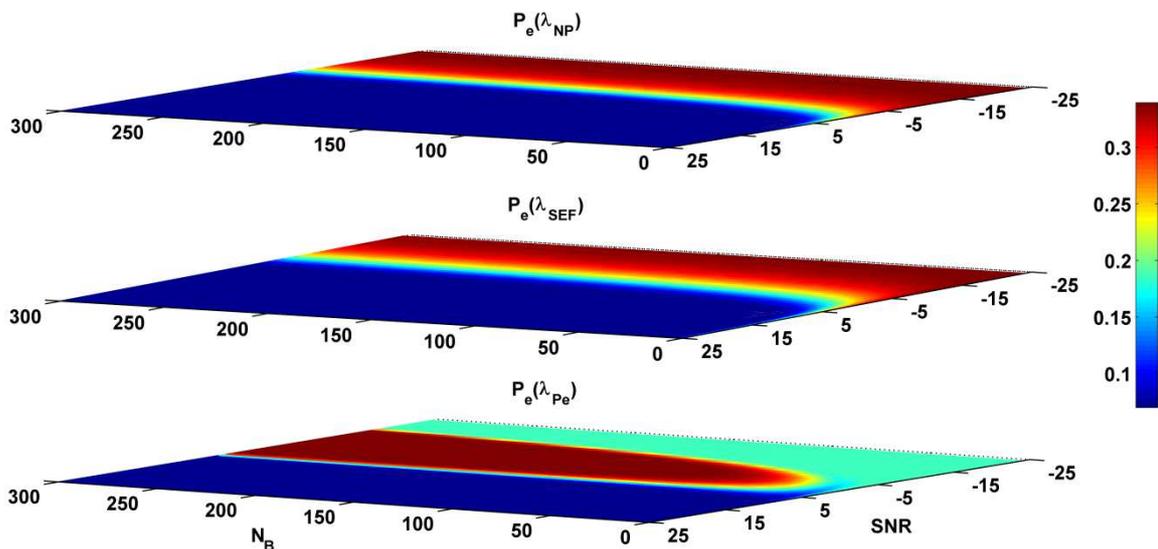


Figure 1. P_e for changing SNR and N_B at $P(h_0)=0.7$ and $P(h_1) = 0.3$

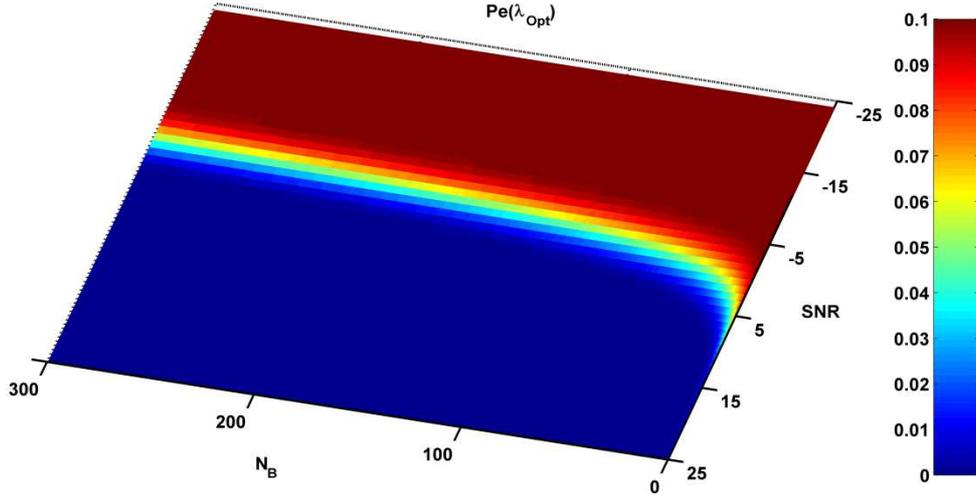


Figure 2. P_e for changing SNR and N_B at $P(h_0)=0.7$ and $P(h_1) = 0.3$ using λ_{opt}

$$P_R(D) = \frac{P(h_1)p(D|h_1)}{P(h_0)p(D|h_0)} \frac{h_0}{h_1} \leq P(\lambda_{opt}) \quad (26)$$

where the posterior ratio threshold would be $P(\lambda_{opt}) = P(h_1|\lambda_{opt})/P(h_0|\lambda_{opt})$. It may be noted that Eqn. (26) reduces to Eqn. (20) for $P(\lambda_{pe}) = 1$ in which case $P(h_1|\lambda_{pe}) = P(h_0|\lambda_{pe})$ as used in deduction of Eqn. (21). In order to achieve simultaneous reduction in the error in detecting joint distributions $p(D, h_i)$, we devise the sensing threshold λ_{opt} by equating the error induced in both cases.

$$\int_{\lambda_{opt}}^{\infty} P(h_0)p(D|h_0)dD = \int_{-\infty}^{\lambda_{opt}} P(h_1)p(D|h_1)dD$$

$$\beta Q\left(\frac{\lambda_{opt} - \mu_{D_0}}{\sigma_{D_0}}\right) = Q\left(-\frac{\lambda_{opt} - \mu_{D_1}}{\sigma_{D_1}}\right) \quad (27)$$

As $\mu_{D_1} > \lambda_{opt} > \mu_{D_0}$ we can achieve an approximation of λ_{opt} by utilizing the Chernoff bound for Q-function and finding the roots of resulting quadratic equation. The arguments of the Q-function in this case are the Mahalanobi's distances of the distributions from the threshold. The Q-function and its Chernoff approximation

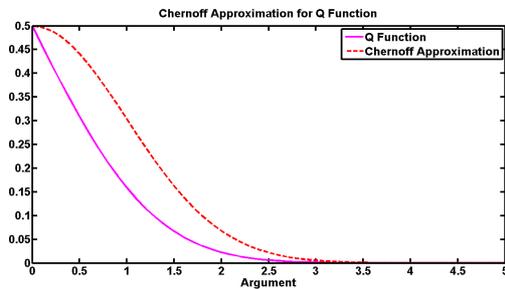


Figure 3. Accuracy of Chernoff Approximation

are plotted in Figure 2. The approximation is highly accurate at very low and very high SNRs and quite reasonable otherwise.

$$\lambda_{opt} \approx \frac{\mu_{D_0} \sigma_{D_1}^2 - \mu_{D_1} \sigma_{D_0}^2}{\sigma_{D_1}^2 - \sigma_{D_0}^2} +$$

$$\frac{\sigma_{D_1} \sigma_{D_0}}{\sigma_{D_1}^2 - \sigma_{D_0}^2} \sqrt{(\mu_{D_1} - \mu_{D_0})^2 + \ln(\beta^2) (\mu_{D_0} \sigma_{D_1}^2 - \mu_{D_1} \sigma_{D_0}^2)} \quad (28)$$

The same equation can be represented in terms of SNR and buffer size. For simplicity we represent the repeating term $2 SNR + 0.5 SNR^2(\gamma - 1) \text{ ass} \hat{r}r$. P_e for this case is given as Eqn. (30) and has been plotted in Figure 3. Clearly the selected threshold results in extensive improvement in P_e reducing the error ceiling considerably.

$$\lambda_{opt} \approx \sigma_z^2 \left[N_B \left(1 - \frac{SNR}{s\hat{r}r} \right) + \sqrt{1 + \frac{1}{s\hat{r}r} \left(2 N_B \ln(\beta^2) + \frac{(N_B SNR)^2}{s\hat{r}r} \right)} \right] \quad (29)$$

The threshold can be prevented from becoming complex by keeping a reasonably large buffer size. For example, a buffer size of 400 samples is sufficient to keep the under root terms as positive for SNR ranging from -25 dB to 25 dB and $P(h_0)$ between 0.1 to 0.9.

$$P_e(\lambda_{opt}) = P(h_0) Q\left(\frac{\lambda_{opt}}{\sigma_z^2 \sqrt{2N_B}} - \frac{\sqrt{N_B}}{2} + P(h_1) \right)$$

$$Q\left(\frac{-1}{\sqrt{2SNR+1}} \left[\frac{\lambda_{opt}}{\sigma_z^2 \sqrt{2N_B}} - \sqrt{\frac{N_B}{2}} (SNR + 1) \right] \right) \quad (30)$$

6. Simulation Results

The simulations have been carried out for simple BPSK case considering zero mean AWGN and varying conditions of $SNR = \mu_{D_1}^2 / \sigma_{D_0}^2$. In order to elaborate the effect of priors three different prior ratios $\beta = P(h_0)/P(h_1)$ have been simulated assuming $P(h_0) = 0.3, 0.5$ and 0.7 . In classical ML case the priors would be equal and β would be unity. However as the prior probability of channel occupation $P(h_1)$ changes in comparison to the probability of channel vacancy $P(h_0)$, effects of changing priors become prominent. The sum of $P(h_1)$ and $P(h_0)$ remains one in all cases. In Figure 4-7 we plot the effects of β while changing SNR on P_e for three different values of β while using the four different sensing thresholds $\lambda_{NP}, \lambda_{SEF}, \lambda_{Pe}$ and λ_{opt} for signal detection.

The β is simulated as 1 (classical ML case with equal priors), 2.33 ($P(h_0) > P(h_1)$) and 0.43 ($P(h_1) > P(h_0)$). The buffer size is fixed at 400. As evident from the results, the changing β has significant bearing on the accuracy of the spectrum sensing algorithm.

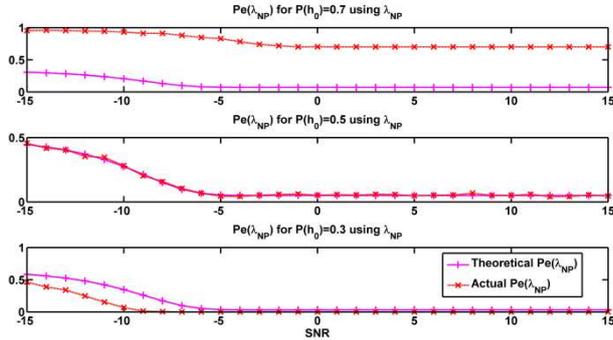


Figure 4. Effect of Changing SNR on P_e for different β using λ_{NP}

The effects of changing β for λ_{NP} are indicated in Figure 4. For the typical ML case where the priors are equal, the P_{md} and P_{fa} follow the calculated theoretical results. However as the ratio of priors becomes un-symmetric the results greatly vary from theoretical values because the effects of changing priors have not been considered while selecting the threshold. There is deterioration in P_e for high $P(h_0)$ because the high prior value for signal distribution and low prior value for noise distribution pushes the distributions closer thus increasing the overlap. This increased overlap results in increased P_e . As the two likelihood distributions have been scaled by different factors, the λ_{NP} is not accurate anymore and cannot ensure a fixed P_{fa} as desired. The situation is especially poor where SNR and $P(h_1)$ are both very low. This is because the noise distribution which already has a large variance (low SNR) is further shifted to the right overlapping the signal distribution due to multiplication with a large prior value. This results in P_{md} and P_{fa} both approaching to unity. This amply elaborates that λ_{NP} would perform extremely poor in case of low channel occupancy conditions. For high $P(h_1)$ the distributions are pushed further apart than the perceived ML case while calculating the threshold. This results in reduced overlap of the two distributions and improvement in P_e .

For λ_{SEF} , the theoretical and simulated results match exactly for equal priors case as reflected in Figure 5. However, the change in priors results in variations in P_e as the priors were not incorporated while selecting the threshold. P_e deteriorates for high $P(h_0)$ and improves for low $P(h_0)$ for similar reasons as discussed in the last paragraph. However the P_e in this case drops to zero while for the λ_{NP} , it could level off to $\alpha P(h_0)$ at best. Thus λ_{SEF} outperforms λ_{NP} but does not cater for changing priors.

Simulation results reveal that the λ_{Pe} is the most robust classical detection technique against changing prior values (Figure 6). The simulated results are quite similar to the theoretical values for maximum SNR range.

There is improvement for high channel occupancy in comparison to the previous two thresholds. For the low $P(h_0)$ the P_e increases.

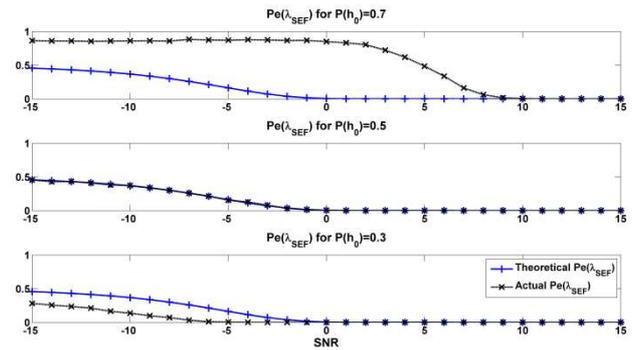


Figure 5. Effect of Changing SNR on P_e for different β using λ_{SEF}

As the point of intersection of the two posterior distributions has been considered as the threshold, the asymmetric distributions imply a drop in P_{fa} while on the other hand a sharp increase in P_{md} . High P_{md} scaled by a high $P(h_1)$ results in overall increase in P_e . This justifies our assumption that the point of intersection of the two posterior distributions is not an optimum threshold for the unequal prior case.

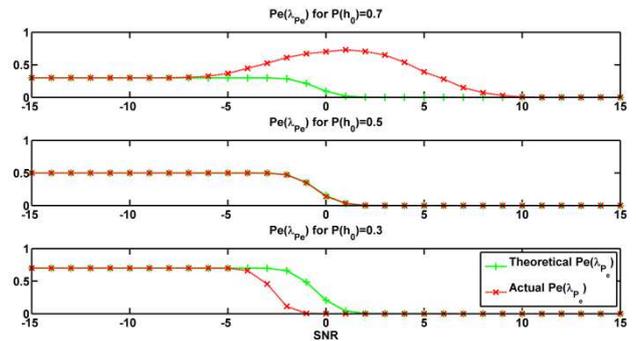


Figure 6. Effect of Changing SNR on P_e for different β using λ_{Pe}

The results for our proposed approach of selecting λ_{opt} as the sensing threshold have been displayed in Figure 7. The theoretical values of P_e do not tally with the simulated results except for equal prior case, as these are based on Chernoff approximation. At high $P(h_0)$, λ_{opt} behaves similar to the λ_{SEF} but as the β approaches to unity, the benefit of using this threshold become obvious. At $\beta = 1$ the λ_{opt} gives exactly the same results as λ_{SEF} . This validates our analysis that instead of considering the point of intersection as threshold, we are aiming to reduce the overall error by simultaneously reducing the P_{fa} and P_{md} in the overlapping posterior distributions (as used in Eqn. 27) by equating them. As the probability of occupation further reduces, the proposed threshold greatly reduces the overall P_e and outperforms the classical approaches with significant margin (13.56 dB, 9.22 dB and 15.44 dB improvement over λ_{NP} , λ_{SEF} and λ_{Pe} respectively at $P(h_0) = 0.3$ and SNR=-15dB).

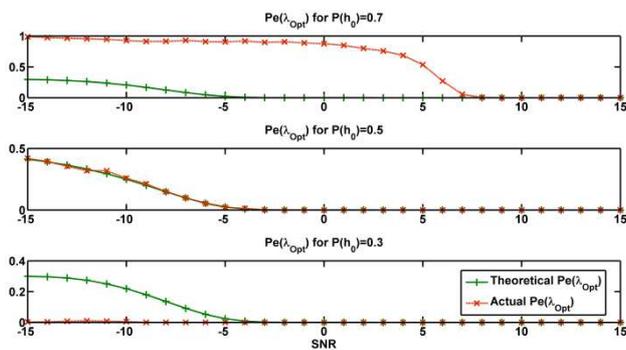


Figure 7. Effect of Changing SNR on P_e for different β using λ_{Opt}

7. Conclusion

The paper aims at quantifying the inadequacies of using classical ML detection for spectrum sensing. The effects of changing prior probabilities of channel occupation or vacancy are simulated and it is concluded that these effects are considerable. The performance degradation is especially aggravated at low SNR values and high variations in prior probabilities. We have also suggested an optimum sensing threshold for minimum P_e which is more robust to variations in priors and gives much reduced P_e in comparison to classical sensing thresholds.

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