Abstract: A novel watermarking algorithm using two layers of transform, first layer is Complex Wavelet Transform and the second one is the singular value decomposition (SVD) is proposed. Image watermarking is to embed copyright data in image bit streams. Our proposed scheme demonstrates the effective and robust of watermarking algorithms using a hybrid of strong transforms. The proposed scheme presenting more security and robustness against attacks. Imperceptibility and robustness were measured and the experimental results were compared to technique using DWT-SVD. The proposed watermarking algorithm is considerably more robust and effective.

Keywords: watermarking, protection, Complex Wavelet Transform, Wavelet Transform, imperceptible, Singular Value Decomposition.

1. Introduction

Techniques using SVD has got popularity. The SVD method has taken attention in image processing, pattern recognition and information security.

One approach of using SVD in watermarking is to apply the SVD to the cover image and modify all the SVs to embed the watermark data. The singular values (SVs) changes very little for most types of attacks which considered the significant property of SVD watermarking.

As found in algebra, an image is a matrix of non-negative entries. If A is a square image, denoted as A ∈ R_{nn}, where R represents the real number domain, then SVD of A is defined as [1]:

\[ A = USV^T \]  

Where U and V are orthogonal matrices such that U^T U = I, V^T V = I, I is an identity matrix. \( S = \text{diag}(\sigma_1, \ldots, \sigma_p) \).

Where, \( p = \min(m, n) \), \( \sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_p \geq 0 \), are the singular values of A. This decomposition is known as the Singular Value Decomposition (SVD) of A, and can be written as follows [1]:

\[ \text{SVD}(A) = [USV] \]  

Or

\[ A = USV^T \]  

The stability of SVs specifies that, when there is a little disruption with A, the deviation of SV’s is not larger than the largest SV of the matrix [1]. Applying SVD to digital image has advantages [2, 3, 4].

i) The most important advantage of singular values of an image is the stability when little perturbation affects the image.

ii) The algebraic features of image can be represented by the SVs, which are not visual.

The dual-tree CWT first introduced by using Kingsbury in 1998 [5]. The concept at the back of the twin-tree technique is simple, it works on real DWTs; 1st DWT offers the actual part of the remodel where 2nd DWT offers the imaginary part. Evaluation and synthesis Feedbacks are used to put in force. DT-CWT and its inverse are shown in Figures (1) and (2). Real wavelet transforms use unique sets of filters. The 2 sets of filters are collectively designed in order that the overall rework is approximately analytic. Permit h0(n) and h1(n) denote the low-bypass/high-pass filter out pair for the upper feedback(fb), and permit g0(n) and g1(n) denote the low-skip/excessive-bypass clear out pair for the decrease Feedbacks (FBs). The result wavelets related to every of the 2 real wavelet transforms are ψh(t) and ψg(t), the filters are designed in order that the complex wavelet \( \psi(t) = H \psi_h(t) \) is analytic. Equivalently, they are designed so that \( \psi(t) \) about the Hilbert remodel of \( \psi_h(t) \) denoted as \( \psi(t) \) = I\Hψ(t).

Notice that the filters are actual; there are no complex mathematics is needed for the implementation of the twin-tree CWT. To invert the rework, the actual and imagined components are reversed. Inverse of actual DWTs are not as actual signals. Those actual alerts are averaged to achieve the ultimate result. Bear in mind x(n) is recovered from each real or imaginary part of a complex number; but, such inverse DT-CWTs do no longer catch advantages associate analytic wave remodel offers. If the real coefficients are delineate through the square matrix. Matrices \( \text{Fh and Fg} \), then the twin-tree CWT could also be delineate by means of the matrix. The DT-CWT is likewise clean to implement. Because of the particular reality there may even be no records drift between the two actual DWTs, they may ever be applied exploitation present DWT code program. Makeover is of course parallelized for inexperienced implementation. Further, owing to the actual fact the DT-CWT is administrated through the usage of 2 actual DWT, the usage of the DT-CWT are often educated through the present plan and exercise of real wavelet transforms. As an example, wavelet-based sign process methods that embody thresholding of wavelet coefficients which are developed for real wavelet transforms can also be administrated to the DT-CWT [5]. It got to be noted, however, that the DT-CWT needs the layout of recent filters. Usually, it needs a combine of separate out units chosen in order that the corresponding wavelets form associate degree approximate Hilbert make over pair. Present filters for wavelet transforms ought to not be accustomed enforce every trees of the DT-CWT. Pairs of Daubechies’ wavelet filters do not satisfy the need that \( \psi(t) = H \psi(t) \). If DT-CWT is applied with filters not pleasing this demand, the process cannot produce the advantages of analytic wavelets outlined.

A Novel Watermarking Technique Based on Hybrid Transforms

Hanaa A. Abdallah 1,2, Sara Alsodairi 1

1 Department of Information Technology, College of Computer and Information Sciences, Princess Nourah Bint Abdulrahman University (PNU), Riyadh, Saudi Arabia

2 Department of Electronics and Communication Engineering, faculty of engineering, Zagazig University, Egypt
Oscillations which create singularity extraction and signal
modeling, specifically, and extremely difficult [5]. It’s
possible for a wave overlapping a singularity to possess little
low or perhaps zero wavelet coefficients. The second
drawback is Shift Variance which complicates wavelet-area
process. To avoid these problems, recollect a smooth signal
\( x(t-t_0) \) like the step function; [5] which analyzed using DWT
with sufficient wide variety of vanishing moments. Its
wavelet coefficients encompass samples of the step reaction
of the wavelet. [5]

\[
d(j, n) \approx 2^{-j/2} \Delta \int_{-\infty}^{\infty} |\psi(t)|dt
\]

(5)

where \( \Delta \) : peak of the jump, \( \psi(t) \) : bandpass function.
\( d(j, n) \) : step response which is a function of \( n \). 2 \( j \) in the
upper limit (\( j \geq 0 \)) amplifies the sensitivity leading to strong
shift variance. The third problem is Aliasing non-ideal low
and high-pass filters which used in (DWT), results in
aliasing. The IDWT cancels this aliasing, but when the
wavelet and scaling coefficients are not changed. The forth
problem is Absence of Directionality; this problem critically
thwarts demonstrating and treating geometric image
structures like ridges and edges.

There is a simple strategy to these 4 DWT shortcomings. The
secret's to note that the Fourier rework does not suffer from
those troubles. Firstly, the significance of the Fourier
transform does no longer oscillate undoubtedly and
negatively however it gives a clean positive cover within the
Fourier field. Secondly, the significance of the Fourier
remodel is impeccably transferring invariant, through an easy
linear section balance coding the shift. Thirdly, the
coefficient of Fourier are not aliased and do now not relies
upon complex aliasing cancellation belongings to reconstruct
the signal. Eventually, the sinusoids of the multi-
Dimensional Fourier basis are notably directional plane
waves. Here , a CWT-based totally SVD watermarking
method for grey and color images is presented . The
watermark embedding method is achieved at the SVs of the
excessive-frequency bands. Watermark embedding in this
paper is applied with the LSB and the additive
methods.

The rest of the paper is organized as follows: Section 2
presents literature review of related work, section 3 explains
the watermark embedding steps for the existed DWT-based
SVD, Section 4 proposed the new robust CWT Based SVD
Watermarking method. Section 5 presents the experimental
results, and comparing the proposed technique with the
existed one and Section 6 concludes the achieved work.

2. Related Work

In the literature, there are several papers on the use of the
DWT with SVD for information hiding in digital images. In
[6], DWT is applied to transform an image into frequency
subbands, and a particular band is modified for data
embedding by dividing the band into blocks of size 4x4.
SVD is applied to every block and a watermark is embedding
in the diagonal of every block. Normalized correlation
coefficient NC is used to measure the degree of similarity
between the original watermark and the extracted watermark
when the watermarked image is attacked. When DWT and
SVD are combined the watermarking method outperforms
the conventional DWT method with respect to robustness to
different attacks. In [7], the method in this paper acts well for
image processing operations, combining DWT with SVD.
method is used to hide the data in the high frequency band of the image. In [8], K level -DWT is applied to image, then SVD is applied to LH and HL and the watermark is embedded in them. But in low (LL) and high frequency (HH) bands, the watermark is embedded using the dispensed Discrete Wavelet transform (DDWT) approach. Two approaches have been examined in the presence of attacks and it was clear that they are robust to the cropping attacks, Gaussian noise, sharpness, histogram equalization, and rotation. In [9], the cover image and the watermark are pre-processed to hide the watermark. The evaluation illustrates that this method is robust to attacks such as cropping and low pass filtering. In [10], a 3L- DWT is applied to an image. All the ten sub bands are transformed using SVD. A new watermarking scheme for images based on the Human Visual System (HVS) characteristics and the SVD in the wavelet domain was discussed in [11]. Results show the performance of the technique according to compression, cropping and scaling attacks. Unfortunately all techniques that use the DWT are not robust to rotation and Gaussian noise attacks.

In [12], a biometrics watermarking technique is proposed using the fractional CWT and SVD. The scheme begins with detecting the proper biometrics of user using scanner. Then transform order generation using SURF and biometrics images. The cover image is randomized using chaotic map and Heisenberg decomposition. And then is embedding in the FrDT-CWT by modifying the singular values of the randomized image. A verification step is presented to affirm the watermarked photograph.

In [13], the technique is proposed using (DTCWT) in low dimensional subspace for gray scale images. Here the cover image is decomposed using 3 level DT-CWT decomposition where large scale features having low frequency coefficients are extracted. The features are then analyzed using the score matrix obtained from principal component analysis (PCA). Next, Singular values from the score matrix are then obtained using the singular value decomposition (SVD) in a lower dimensional sub space. The watermark is subjected to similar processing after being scrambled using Arnold transform.

In [14], satellite image is enhanced using three wavelet techniques: discrete wavelet transforms (DWT) - Stationary wavelet transforms (DWT-SWT) and Dual tree complex wavelet transforms (DT-CWT). The result shows that DT-CWT highlight superior performance.

In [15], the deep hiding and extracting algorithms is proposed with modified least significant bit and artificial bee colony with the multi levels steganography approach using a recursive loop and lossless image compression to hide the data in unlimited levels. Two layers of randomization approach are applied with Dynamic encryption algorithm based on AES using different key in each level. The Artificial bee colony algorithm is used to get high imperceptible and smoother image. The experimental results show the effectiveness of this algorithm for hiding large data up to four bits per pixel with less noise comparing to different technique.

3. Watermarking using DWT and SVD

The first proposed scheme is applied to the DWT coefficients by modifying the SV’s.

3.1 Watermark Embedding

The watermark embedding steps for DWT-based SVD watermarking are summarized as follows.

1. Estimate the 2-level DWT of the image A. This generates seven high –frequency DWT sub-bands; LL2, HL2, LH2, HH2, LH1, HL1, HH1.
2. The SVD is applied to the HL2 sub-band. This decomposes the sub-band coefficient matrix into three independent matrices:

\[ A_{HL2} = U * S * V^T \]  

3. The 1-level DWT is applied to the watermark W.
4. Apply the SVD operation on the HL2 of the watermark.

LSB manner:
5. Embed the values of S_w after binarization into S by substituting the S_w bits into the LSB of S to get S’.

Additive manner:
5. Embed the watermark as follows:

\[ S’ = S + k * S_w \]  

To complete the embedding process in both manners, the inverse SVD is applied to the modified S matrix to get the modified matrix A_{HL2}. As follows:

\[ A_{HL2}’ = U * S’ * V^T \]  

6. Apply the IDWT on the matrix A_{HL2}’. To generate watermarked image A_w.

3.2 Watermark Extraction

The watermark image can be extracted from the watermarked image (A_w) in the following steps:

1. Decompose the watermarked image using DWT and SVD as in steps 1 to 3 of the embedding process.
2. The 2-level DWT of the watermarked image A_w is computed.
3. Apply the SVD operation on the HL2 subband of the watermarked image A_w. The SVD operation decomposes the subbands coefficient matrix into three matrices:

\[ A_{wHL2} = U_w * S_w * V_w^T \]  

LSB manner
4. Extract the embedded singular values of the watermark from the diagonal matrix S_w as follows:

\[ S_{out} = \text{LSB} (S_w) \]  

Additive manner:
5. Extract the matrix of singular values of the watermarked image as follows.

\[ S_{out} = (S_w - S) / k \]  

Construct the DWT coefficients by using the SV’s matrix S_{out}, which calculated in equation (11) and (U, V) which are generated in embedding process.

\[ W_{rec} = U_w * S_{out} * V_w^T \]  

Where W_{rec} is the matrix of the recovered watermark in the wavelet domain.
6. Finally, apply the IDWT to W_{rec} to construct the visual watermarks.

In transform domain watermarking, However DWT results tend to be disappointing [16, 17], it still used in researches. In general, the DWT produces watermarks with the quality visible excellent due to the absence of blockading artifacts. But, it has two drawbacks; lack of shift invariance and bad directional selectivity for diagonal capabilities [18, 19]. An essential latest development in wavelet-related studies is the layout and implementation of two-D multiscale transforms that represent edges extra efficiently than does the DWT.
4. The proposed CWT based SVD Watermarking

The proposed scheme is performed by the modification of SVs of the high frequency subbands of images with either the LSB method or the addition method.

4.1 Proposed Method with the LSB Method.

4.1.1 Embedding the watermark

The inserting of watermark is shown using the block diagram in Figure 4.
1) 1-level of complex wavelet transform is applied to the watermarked image.
2) Then SVD transform is applied to all high-pass bands.
3) Process the original image using the DT-CWT and SVD.
4) The 1-level CWT is computed for the whole image. This method produces six high frequency sub-bands.
5) SVD is applied to high-frequency subband: 
   \[ A_j = U_j \ast S_j \ast V_j^T, \quad j = 1, \ldots, 6 \]  \hspace{1cm} (13)
   
   \[ j: \text{high-pass subbands.} \]
   \[ S_j: \text{SVs matrix.} \]
6) Modify the SVs of high-pass subband with SV’s of watermark using LSB method:
7) Embed the values of SVs of the CWT coefficients of the watermark after binaryization into the LSB of the SVs of the original image.
8) Acquire the 6 subbands of the new DT-CWT coefficients:
   \[ \hat{A}_j = U_j \ast \hat{S}_j \ast V_j^T \]  \hspace{1cm} (14)
9) At last long, apply the reverse CWT utilizing the altered CWT coefficients. This activity creates the last watermarked image.

4.1.2 Extracting the watermark.

The watermark is extracted without using the cover image. The extraction process is depicted below:
1) Compute the 1-level DT-CWT for the image. This produce 6 high-frequency sub-bands.
2) SVD is applied to every high-pass subband.
   \[ \hat{A}_j = U_j \ast \hat{S}_j \ast V_j^T \]  \hspace{1cm} (15)
   
   \[ j = 1, \ldots, 6 \]
3) The SVs are extracted from each high frequency bands. Take the LSB of each coefficient of the SVs of the watermarked image.
4) Using the matrix \( S_j^W \) of the watermarked image and the vectors \( U_{w,j}, V_{w,j} \) generated in the embedding process to generate the DT-CWT coefficients of all high-frequency bands. 
   \[ A_{w,j} = U_{w,j} \ast S_j^W \ast V_{w,j}^T, \quad j = 1, \ldots, 6. \]  \hspace{1cm} (16)
5) At long last, backwards transform is applied to develop the watermarks.

4.2 Proposed Method with the Additive Method.

4.2.1 Embedding the watermark

The process of embedding is shown in Figure 5
1) 1-level of CWT is applied to watermark.
2) SVD is applied to every high-pass band.
3) Process the original image using the DT-CWT and SVD.
4) 2-level CWT is compute. This method generates six high-frequency sub-bands.
5) Apply the singular value decomposition to every high-frequency subband:

\[ A_j = U_j \ast S_j \ast V_j^T, \quad j = 1, \ldots, 6 \]  \hspace{1cm} (17)

\[ J: \text{high-pass subbands of the 2-level decomposition,} \]
\[ S_j: \text{SVs matrix.} \]
6) Modify the SVs of every high-frequency subband with the SV’s of the watermark using the additive method.
7) Acquire the 6 subbands:
   \[ \hat{S}_j = S_j + k \ast S_w \] \hspace{1cm} (18)
8) At last long, the opposite DT-CWT is applied to create the last watermarked image.

4.2.2 Watermark Extraction

1) 2-level DT-CWT is applied to the image. This produce 6 high-frequency subbands.
2) SVD is applied to each high-frequency subband.
   \[ \hat{A}_j = U_j \ast \hat{S}_j \ast V_j^T \]  \hspace{1cm} \text{as} \hspace{0.5cm} j = 1, 2, \ldots, 6 \hspace{1cm} (19)
3) the SVs are extracted from every high-frequency subband
4) Using SV framework to develop the DT-CWT coefficients of all six high-frequency bands.
   of the watermarked image and the vectors figured at the time of the inserting procedure
   \[ A_{w,j} = U_{w,j} \ast S_j^W \ast V_{w,j}^T, \quad j = 1, 2, \ldots, 6. \]  \hspace{1cm} (21)
5) At long last, apply the opposite DT-CWT to build the visual watermarks.

5. Simulation results

5.1. DT-CWT based SVD Watermarking Using the Additive Method.
Here, experiments are carried out to test the DT-CWT-based SVD watermarking using the Additive Method. In table (1)
the original images used for the experiments are shown in the first, the different extracted watermarks without attacks and the different extracted watermarks under different attacks also shown. Table (2) shows the high correlation even after the effect of attacks. Both tables indicate the robustness of the proposed method to attacks, and Table (3) shows a comparison in the PSNR between the proposed method and the existing methods. These results reveal the superiority of the proposed DT-CWT-based SVD watermarking method using the additive method to the traditional methods under attacks.

Rotation attack with different angels has been applied to the watermarked image and the robustness has been studied as shown in table 1 and figure 6 (a). The correlation values indicate clearly the robustness of the proposed method as compared to other methods. The DWT-based watermarking is more robust than the proposed method for a rotation with angle -90. A Gaussian noise was added with different variances of 0.006, 0.01, 0.08, 0.1 and 0.5 to the watermarked images. As shown in table 1 and Figure 6(b) values of correlation specify the robustness of the proposed scheme to attack of addition Gaussian noise compared to other methods. Then the image was resized, as shown in table 1 and Figure 6 (c). The correlation values indicate clearly the robustness of the proposed scheme compared to the existing methods. The watermarked image was compressed with quality Q=50%. As shown in table (1) Correlation values clearly indicate the strength of the method under the attack of image compression compared to current methods. We find as shown in table 1 and Figure 6 (d), that the effects of cropping attacks to the watermarked images are limited.

![Figure 6. Behavior of the correlation coefficient between the original watermark and extracted watermark for the different attacks. (a) Rotation. (b) Gaussian noise. (c) Resizing. (d) Cropping.](image)

### 5.2. DT-CWT based SVD Watermarking Using the LSB method.

Here, results of applying the CWT-based SVD watermarking using LSB method are introduced. In table (4) the original images are shown in the first, the different extracted watermarks without attacks and the different extracted watermarks under different attacks also shown. Table (5) shows Comparison between the proposed method and the traditional methods under attacks for Lena image, and Table (6) represents a comparison using PSNR between the proposed method and the existing methods. Both tables clearly specify the robustness of the proposed method to different attacks. These results reveal the superiority of the proposed DWT-based SVD watermarking method using the LSB method to the traditional methods under attacks.

Rotation attack with different angels has been applied to the watermarked image and the robustness has been studied as shown in table 4 and figure 7 (a). The correlation values indicate the robustness of our method as compared to other methods. The DWT-based watermarking is more robust than the proposed method for a rotation with angle -90. A Gaussian noise was added with different variances of 0.006, 0.01, 0.08, 0.1 and 0.5 to the watermarked images. As shown in table 4 and Figure 7(b) the values of correlation specify the large robustness of the proposed technique to the effect of addition of Gaussian noise compared to other methods. Then the image was resized, as shown in table 4 and Figure 7 (c). The correlation values indicate clearly the robustness of the proposed method compared to the existing methods. The watermarked image was compressed with quality Q=50%. As shown table (4) the correlation values indicate clearly robustness of the method against the image compression comparing to existing methods. We find as shown in table 4 and Figure 7 (d), that the effects of cropping attacks to the watermarked images are limited.

![Figure 7. Behaviors of the correlation coefficient of watermark Using LSB under the effect of different attacks. (a) Rotation. (b) Gaussian noise. (c) Resizing. (d) Cropping.](image)

### 6. Conclusions

A novel technique for watermarking the images to confirm copyright of is proposed. The technique is applied by using hybrid of transforms, DWT and SVD using two strategies of embedding LSB and additive methods. The two schemes are coordinated to mishandle their features that are catching of semi-universal features and geometric information of image data by vital components of SVD and spatial frequency restriction of DT-CWT. Results showed the strength of the proposed scheme because it successfully extracted the watermark from images within the absence of the original image. The calculated correlation is extremely near to one which implies the equality of the extracted and original watermarks. Compared to the techniques using just SVD or DWT or DWT-SVD.

### Acknowledgment

This research was funded by the Deanship of Scientific Research at Princess Nourah Bint Abdulrahman University through the Fast-Track Research Funding Program.

### References


321

International Journal of Communication Networks and Information Security (IJCNIS) Vol. 12, No. 3, December 2020


Table 1. Results of Watermarked image using DT-CWT based SVD and the DWT based SVD watermarking technique using additive procedure.

<table>
<thead>
<tr>
<th>Attack type</th>
<th>Watermarked image</th>
<th>Recovered watermarks</th>
<th>Attack type</th>
<th>Watermarked image</th>
<th>Recovered watermarks</th>
<th>Attack type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td><img src="image_url" alt="Original image" /></td>
<td><img src="image_url" alt="Recovered watermark" /></td>
<td>Original</td>
<td><img src="image_url" alt="Original image" /></td>
<td><img src="image_url" alt="Recovered watermark" /></td>
<td>Original</td>
</tr>
<tr>
<td>Without attack</td>
<td><img src="image_url" alt="Watermarked image" /></td>
<td><img src="image_url" alt="Recovered watermark" /></td>
<td>Without attack</td>
<td><img src="image_url" alt="Watermarked image" /></td>
<td><img src="image_url" alt="Recovered watermark" /></td>
<td>Without attack</td>
</tr>
<tr>
<td>Rotated with 90°</td>
<td><img src="image_url" alt="Watermarked image" /></td>
<td><img src="image_url" alt="Recovered watermark" /></td>
<td>Rotated with 90°</td>
<td><img src="image_url" alt="Watermarked image" /></td>
<td><img src="image_url" alt="Recovered watermark" /></td>
<td>Rotated with 90°</td>
</tr>
<tr>
<td>Gaussian noise</td>
<td><img src="image_url" alt="Watermarked image" /></td>
<td><img src="image_url" alt="Recovered watermark" /></td>
<td>Gaussian noise</td>
<td><img src="image_url" alt="Watermarked image" /></td>
<td><img src="image_url" alt="Recovered watermark" /></td>
<td>Gaussian noise</td>
</tr>
<tr>
<td>variance=0.01</td>
<td><img src="image_url" alt="Watermarked image" /></td>
<td><img src="image_url" alt="Recovered watermark" /></td>
<td>variance=0.01</td>
<td><img src="image_url" alt="Watermarked image" /></td>
<td><img src="image_url" alt="Recovered watermark" /></td>
<td>variance=0.01</td>
</tr>
<tr>
<td>Resizing 512-256-</td>
<td><img src="image_url" alt="Watermarked image" /></td>
<td><img src="image_url" alt="Recovered watermark" /></td>
<td>Resizing 512-256-</td>
<td><img src="image_url" alt="Watermarked image" /></td>
<td><img src="image_url" alt="Recovered watermark" /></td>
<td>Resizing 512-256-</td>
</tr>
<tr>
<td>512</td>
<td><img src="image_url" alt="Watermarked image" /></td>
<td><img src="image_url" alt="Recovered watermark" /></td>
<td>512</td>
<td><img src="image_url" alt="Watermarked image" /></td>
<td><img src="image_url" alt="Recovered watermark" /></td>
<td>512</td>
</tr>
<tr>
<td>Compression attack</td>
<td><img src="image_url" alt="Watermarked image" /></td>
<td><img src="image_url" alt="Recovered watermark" /></td>
<td>Compression attack</td>
<td><img src="image_url" alt="Watermarked image" /></td>
<td><img src="image_url" alt="Recovered watermark" /></td>
<td>Compression attack</td>
</tr>
<tr>
<td>Q=50%</td>
<td><img src="image_url" alt="Watermarked image" /></td>
<td><img src="image_url" alt="Recovered watermark" /></td>
<td>Q=50%</td>
<td><img src="image_url" alt="Watermarked image" /></td>
<td><img src="image_url" alt="Recovered watermark" /></td>
<td>Q=50%</td>
</tr>
<tr>
<td>Cropping</td>
<td><img src="image_url" alt="Watermarked image" /></td>
<td><img src="image_url" alt="Recovered watermark" /></td>
<td>Cropping</td>
<td><img src="image_url" alt="Watermarked image" /></td>
<td><img src="image_url" alt="Recovered watermark" /></td>
<td>Cropping</td>
</tr>
</tbody>
</table>

(A) Dt-Cwt-Svd
Results of Watermarked image with the proposed DT-CWT based SVD using the Additive Method, PSNR= 74.6 dB of watermarked image without attack

(B) Dwt-Svd
Results of Watermarked image with the DWT based SVD watermarking method, PSNR= 65.96 dB of watermarked image without attack
Table 2. The proposed watermarking method versus the traditional methods under attacks.

<table>
<thead>
<tr>
<th>Correlation (cr)</th>
<th>DWT</th>
<th>SVD</th>
<th>DWT based SVD</th>
<th>Proposed DT-CWT based SVD</th>
</tr>
</thead>
<tbody>
<tr>
<td>cr1</td>
<td>0.95</td>
<td>0.99</td>
<td>0.996</td>
<td>0.996</td>
</tr>
<tr>
<td>cr2</td>
<td>0.96</td>
<td>0.98</td>
<td>0.996</td>
<td>0.9997</td>
</tr>
</tbody>
</table>

Table 3. Comparison between the proposed watermarking method and the traditional methods without attacks.

<table>
<thead>
<tr>
<th>Method</th>
<th>PSNR (dB)</th>
<th>Run time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DWT</td>
<td>65.96</td>
<td>5.7</td>
</tr>
<tr>
<td>SVD</td>
<td>33.37</td>
<td>24.8</td>
</tr>
<tr>
<td>DWT based SVD</td>
<td>65.96</td>
<td>2.2</td>
</tr>
<tr>
<td>Proposed DT-CWT based SVD</td>
<td>74.6</td>
<td>1.6</td>
</tr>
</tbody>
</table>
Table 4. Results of Watermarked image with the proposed DT-CWT based SVD and with the DWT based SVD watermarking method using LSB method.

| Attack type     | Watermarked image | Recovered watermarks | (A) Dt-cwt-Svd  
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td><img src="image1" alt="Original Image" /></td>
<td><img src="image2" alt="Recovered Watermarks" /></td>
<td>Results of Watermarked image with the proposed DT-CWT based SVD Using LSB Method, PSNR=60.65dB</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Without attack</td>
<td><img src="image3" alt="Without Attack Image" /></td>
<td><img src="image4" alt="Recovered Watermarks" /></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rotated with 90°</td>
<td><img src="image5" alt="Rotated 90° Image" /></td>
<td><img src="image6" alt="Recovered Watermarks" /></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gaussian noise variance=0.01</td>
<td><img src="image7" alt="Gaussian Noise Image" /></td>
<td><img src="image8" alt="Recovered Watermarks" /></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Resizing 512x512</td>
<td><img src="image9" alt="Resizing Image" /></td>
<td><img src="image10" alt="Recovered Watermarks" /></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Compression attack with Q=50%</td>
<td><img src="image11" alt="Compression Image" /></td>
<td><img src="image12" alt="Recovered Watermarks" /></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cropping</td>
<td><img src="image13" alt="Cropping Image" /></td>
<td><img src="image14" alt="Recovered Watermarks" /></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Watermarked image</th>
<th>Recovered watermarks</th>
<th>Attack type</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image15" alt="Original Image" /></td>
<td><img src="image16" alt="Recovered Watermarks" /></td>
<td>Original</td>
</tr>
<tr>
<td><img src="image17" alt="Without Attack Image" /></td>
<td><img src="image18" alt="Recovered Watermarks" /></td>
<td>Without attack</td>
</tr>
<tr>
<td><img src="image19" alt="Rotated 90° Image" /></td>
<td><img src="image20" alt="Recovered Watermarks" /></td>
<td>Rotated with 90°</td>
</tr>
<tr>
<td><img src="image21" alt="Gaussian Noise Image" /></td>
<td><img src="image22" alt="Recovered Watermarks" /></td>
<td>Gaussian noise variance=0.01</td>
</tr>
<tr>
<td><img src="image23" alt="Resizing Image" /></td>
<td><img src="image24" alt="Recovered Watermarks" /></td>
<td>Resizing 512x512</td>
</tr>
<tr>
<td><img src="image25" alt="Compression Image" /></td>
<td><img src="image26" alt="Recovered Watermarks" /></td>
<td>Compression attack with Q=50%</td>
</tr>
<tr>
<td><img src="image27" alt="Cropping Image" /></td>
<td><img src="image28" alt="Recovered Watermarks" /></td>
<td>Cropping</td>
</tr>
</tbody>
</table>
Table 5. Comparison between the proposed methods and the traditional methods under attacks for Lena image.

<table>
<thead>
<tr>
<th>Method</th>
<th>Correlation (cr)</th>
<th>DWT</th>
<th>DWT based SVD</th>
<th>Proposed DT-CWT based SVD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>cr1</td>
<td>cr2</td>
<td>cr1</td>
</tr>
<tr>
<td>No attack</td>
<td></td>
<td></td>
<td></td>
<td>0.9961</td>
</tr>
<tr>
<td>Rotation angle</td>
<td></td>
<td></td>
<td></td>
<td>0.9960</td>
</tr>
<tr>
<td>-90</td>
<td>0.8605</td>
<td>0.7555</td>
<td>0.9962</td>
<td>0.9929</td>
</tr>
<tr>
<td>-1</td>
<td>0.7199</td>
<td>0.6942</td>
<td>0.9944</td>
<td>0.9945</td>
</tr>
<tr>
<td>5</td>
<td>0.7184</td>
<td>0.6993</td>
<td>0.9956</td>
<td>0.9948</td>
</tr>
<tr>
<td>30</td>
<td>0.7178</td>
<td>0.7045</td>
<td>0.9951</td>
<td>0.9939</td>
</tr>
<tr>
<td>120</td>
<td>0.7179</td>
<td>0.7073</td>
<td>0.9950</td>
<td>0.9940</td>
</tr>
<tr>
<td>Noise variance</td>
<td></td>
<td></td>
<td></td>
<td>0.9948</td>
</tr>
<tr>
<td>0.006</td>
<td>0.6756</td>
<td>0.6342</td>
<td>0.9916</td>
<td>0.9900</td>
</tr>
<tr>
<td>0.01</td>
<td>0.6734</td>
<td>0.6305</td>
<td>0.9911</td>
<td>0.9900</td>
</tr>
<tr>
<td>0.08</td>
<td>0.6704</td>
<td>0.6275</td>
<td>0.9911</td>
<td>0.9900</td>
</tr>
<tr>
<td>0.1</td>
<td>0.6700</td>
<td>0.6263</td>
<td>0.9900</td>
<td>0.9810</td>
</tr>
<tr>
<td>Resizing ratio</td>
<td></td>
<td></td>
<td></td>
<td>0.9946</td>
</tr>
<tr>
<td>0.1</td>
<td>0.9333</td>
<td>0.9151</td>
<td>0.9994</td>
<td>0.9986</td>
</tr>
<tr>
<td>0.4</td>
<td>0.8332</td>
<td>0.8063</td>
<td>0.9971</td>
<td>0.9965</td>
</tr>
<tr>
<td>0.5</td>
<td>0.9981</td>
<td>0.9955</td>
<td>0.9981</td>
<td>0.9955</td>
</tr>
<tr>
<td>0.7</td>
<td>0.7693</td>
<td>0.7207</td>
<td>0.9954</td>
<td>0.9944</td>
</tr>
<tr>
<td>0.9</td>
<td>0.8130</td>
<td>0.7269</td>
<td>0.9954</td>
<td>0.9945</td>
</tr>
<tr>
<td>Compression quality</td>
<td></td>
<td></td>
<td></td>
<td>0.9963</td>
</tr>
<tr>
<td>10%</td>
<td>0.8539</td>
<td>0.7776</td>
<td>0.9945</td>
<td>0.9942</td>
</tr>
<tr>
<td>30%</td>
<td>0.7818</td>
<td>0.7320</td>
<td>0.9947</td>
<td>0.9941</td>
</tr>
<tr>
<td>50%</td>
<td>0.7504</td>
<td>0.7004</td>
<td>0.9949</td>
<td>0.9944</td>
</tr>
<tr>
<td>Cropping ratio</td>
<td></td>
<td></td>
<td></td>
<td>0.9960</td>
</tr>
<tr>
<td>512-500</td>
<td>0.8825</td>
<td>0.7628</td>
<td>0.9945</td>
<td>0.9940</td>
</tr>
<tr>
<td>512-400</td>
<td>0.8825</td>
<td>0.7628</td>
<td>0.9947</td>
<td>0.9933</td>
</tr>
<tr>
<td>512-300</td>
<td>0.8956</td>
<td>0.7855</td>
<td>0.9943</td>
<td>0.9929</td>
</tr>
</tbody>
</table>

Table 6. Comparison between the proposed watermarking method and the traditional methods without attacks using LSB method for Lena image

<table>
<thead>
<tr>
<th>Method</th>
<th>PSNR (dB)</th>
<th>Run time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DWT</td>
<td>35.90</td>
<td>2.53</td>
</tr>
<tr>
<td>DWT based SVD watermarking</td>
<td>45.77</td>
<td>3.9</td>
</tr>
<tr>
<td>Proposed DT-CWT based SVD watermarking</td>
<td>60.65</td>
<td>7.125</td>
</tr>
</tbody>
</table>