Performance Analysis of Unreliable Sensing for an Opportunistic Spectrum Sharing System

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Abstract: Opportunistic spectrum sharing (OSS) is a promising technique to improve spectrum utilization using cognitive radios. Unreliable spectrum sensing by cognitive radios is inevitable in the OSS system. In this paper, we analyze the types of unreliable sensing and their impact on the system performance. The secondary users equipped with cognitive radios sense channels that are unused by the primary users and opportunistically make use of them without causing harmful interference to the primary users [1]. In such a scenario of opportunistic spectrum sharing, there are two types of wireless networks. The one that owns the license for spectrum usage is referred to as the primary system, its users are referred to as the primary users, and the calls generated from primary users constitute the primary traffic (PT) stream. The other network in the same service area is referred to as the secondary system, its users are referred to as the secondary users, which opportunistically shares the spectrum resource with the primary users. The calls generated from the secondary users constitute the secondary traffic (ST) stream. The system consisting of the primary and secondary systems is called an OSS system. By allowing secondary users to reclaim idle channels, much higher spectrum efficiency can be achieved, even under unreliable spectrum sensing [2].

In the OSS wireless system, an initiating secondary user senses a channel is idle and then makes use of such a channel. Similarly, an ongoing secondary user also detects when a primary user accesses its channel and then either moves to another idle channel or moves to a buffer if no idle channel is available. Unreliable spectrum sensing is modeled by false alarm and misdetection events for both initiating and ongoing secondary users. We solve the steady-state probability vector of the system and derive a set of performance metrics of interest. Numerical results are presented to highlight the analysis. The proposed modeling method can be used to evaluate the performance of future opportunistic spectrum sharing networks.

Keywords: Opportunistic spectrum sharing, Primary users, Secondary users, Unreliable sensing, False Alarm, Misdetection, Markov process.

1. Introduction

Opportunistic spectrum sharing (OSS) is a promising technique to improve spectrum utilization using cognitive radios. Cognitive radios are capable of sensing idle frequency channels and opportunistically make use of them without causing harmful interference to the primary users [1]. In such a scenario of opportunistic spectrum sharing, there are two types of wireless networks. The one that owns the license for spectrum usage is referred to as the primary system, its users are referred to as the primary users, and the calls generated from primary users constitute the primary traffic (PT) stream. The other network in the same service area is referred to as the secondary system, its users are referred to as the secondary users, which opportunistically shares the spectrum resource with the primary users. The calls generated from the secondary users constitute the secondary traffic (ST) stream. The system consisting of the primary and secondary systems is called an OSS system. By allowing secondary users to reclaim idle channels, much higher spectrum efficiency can be achieved, even under unreliable spectrum sensing [2].

In the OSS wireless system, an initiating secondary user senses a channel is idle and then makes use of such a channel. Similarly, an ongoing secondary user also detects when a primary user accesses its channel and then either moves to another idle channel, if one is available, or moves to a waiting pool. In the latter case, the secondary user's call waits in a buffer until either a new channel becomes available or a predefined maximum waiting time expires. The reliable detection of primary users is a major challenge for the implementation of an OSS system. The spectrum usage of the secondary users is contingent on the requirement that the interference to the primary users must be limited to a certain threshold.

Much research about opportunistic spectrum sharing or dynamic spectrum access has been developed in the past a few years. In [3], collaborative spectrum sensing was proposed and studied as a means to combat the shadowing or fading effects that a user experiences. In [4], a measurement-based model was proposed to statistically describe the busy and idle periods of a wireless LAN. An energy-based detection and a feature-based detection were explored to identify spectrum opportunities. In [5], a multi-channel MAC protocol was developed to enable the interoperation of the primary-secondary overlay network. In [6], a sensing-based approach was studied for channel selection in spectrum-agile communication systems. In [7], a multichannel OFDMA technique was proposed for networks allowing opportunistic spectrum access (OSA). The OSA nodes compete amongst themselves and with the primary users by using fast retrials. In [8], an admission control algorithm in conjunction with a power control scheme was proposed for cognitive wireless networks such that quality-of-service requirements of all admitted secondary users are satisfied. In [9], a collaborative scheme was developed for a group of frequency agile radios to estimate the maximum interference-free transmit power (MIFTP) without causing harmful interference to the primary receivers. In this paper\textsuperscript{1}, we model an opportunistic spectrum sharing system under unreliable spectrum sensing and evaluate its performance. Some earlier research about the OSS system has been done under perfect sensing [11] and unreliable sensing with only initiating secondary users involved [2]. In [11], no sensing errors are considered for secondary users. In [2], the sensing errors only occurred for initiating secondary users. That is, the secondary user that is searching an idle channel may make a sensing error. We propose in this paper that not only the initiating secondary users but also the ongoing secondary users may make sensing errors. We introduce multiple parameters, such as probabilities of type I and type II false alarm, and probabilities of class-A and class-B misdetection, to describe these sensing errors, which brings more flexibility for the performance evaluation of an OSS system.

The remainder of the paper is organized as follows.

\textsuperscript{1}An early version of this work was presented in part at the 2010 International Conference on Networking, Sensing and Control (ICNSC2010), Apr. 2010, Chicago, IL, USA [10].
Section 2 describes the system model and unreliable sensing in further detail. Section 3 develops a Markov model of the system dynamics. Section 4 derives the performance metrics of interest. Section 5 presents numerical results. Finally, the paper is concluded in Section 6.

2. System Model

In the OSS system, the PT calls operate as if there are no ST calls in the system. When a PT call arrives to the system, it occupies a free channel if one is available; otherwise, it will be blocked. Note that a channel being used by an ST call is still seen as an idle channel by the primary network, since here the primary network and secondary network are supposed not to exchange information. Secondary users detect the presence or absence of signals from primary users and maintain records of the channel occupancy status. The detection mechanism may involve collaboration with other secondary users and/or an exchange with an associated base station (BS).

Secondary users opportunistically access the channels that are in idle status. If an initiating secondary call finds an idle channel, it can make use of the channel. If all channels are busy, the secondary call is blocked and considered lost from the system.

When an ongoing secondary user detects or is informed (by its BS or other secondary users) of an arrival of PT call in its current channel, it immediately leaves the channel and switches to an idle channel, if one is available, to continue the call. If at that time all the channels are occupied, the ST call is placed into a buffer located at its BS (for an infrastructured network) or a virtual queue (for an infrastructureless network). The queued ST calls are served in first-come first-served (FCFS) order. The head-of-line (HOL) ST call is reconnected to the system when a channel becomes available before a predefined maximum waiting time expires. Intuitively, the maximum waiting time of an ST call should equal to its residence time in the given service area, if the effect of impatience of the queued ST calls is not considered.

In the ideal case, the quality-of-service experienced by primary users is not affected by the secondary users. However, in practice, a primary call that is actively using a given channel may experience disruption if an initiating ST call searching for a free channel incorrectly determines that the given channel is idle. A second class of disruption events to a primary call may occur when an ongoing secondary user on a given channel fails to detect the presence of an arriving primary user on that channel. We refer to such detection errors as class-A and class-B misdetection events, respectively. Both misdetection events can negatively impact the performance of the primary system. When a misdetection event occurs, both the secondary user and the primary user are using the same channel, causing large interference (large "noise") for each other. One of the results is that both users are dropped from the channel.

On the other hand, an initiating secondary user may incorrectly determine that a channel is busy when in fact the channel is idle. In addition, an ongoing secondary user may also incorrectly determine the presence of a primary user on its channel when in fact no primary user enters the channel. We refer to the former type of error as type I false alarm event, and the latter as type II false alarm event. A false alarm event does not incur performance degradation on the primary system, but lowers the potential spectrum utilization of the OSS system. We denote the probabilities of misdetection class-A and class-B by \( p_a \) and \( p_b \), and the probabilities of type I and type II false alarm by \( p_f \) and \( p_{f2} \), respectively. Hence, the critical point of design and implementation of an OSS system is to keep the unreliable detection probabilities as small as possible so that the caused interference (particularly to the primary users) is restricted in a predefined threshold.

3. Performance Analysis

Suppose the spectrum in the service area is divided into \( N \) channels serving the two types of traffic: primary traffic (PT) and secondary traffic (ST). Arrivals of the PT and ST calls are assumed to form independent Poisson processes with rates \( \lambda_1 \) and \( \lambda_2 \), respectively. The channel occupancy times of the PT and ST calls are assumed to be exponentially distributed with means \( 1/\mu_1 \) and \( 1/\mu_2 \), respectively. The residence time for the ST calls in the service area is assumed to be exponentially distributed with mean \( 1/r_2 \). These assumptions have been found to be reasonable as long as the number of users is much more than that of the channels in a service area, and have been widely used in the literature [12]-[14]. We further assume that both types of traffic occupy one channel per call for simplicity. However, the analysis method used here can be extended to handle variable bandwidth requests (cf. [15]).

Let \( X_1(t) \) denote the number of PT calls in the OSS system at time \( t \). Similarly, let \( X_2(t) \) be the number of ST calls in the system at time \( t \), including the ST calls being served and those waiting in the buffer. The process \((X_1(t), X_2(t))\) is a two-dimensional Markov process with state space

\[
S = \{(n_1, n_2) | 0 \leq n_1, n_2 \leq N\}.
\]

We classify the channel occupancy of the system in state \((n_1, n_2)\) as pre-full (cf. Fig. 1) if \( n_1 + n_2 < N \), as just-full (cf. Fig. 2) if \( n_1 + n_2 = N \), and as post-full (cf. Fig. 3) if \( n_1 + n_2 > N \).

Due to unreliable spectrum sensing, type I and type II false alarm events as well as class-A and class-B misdetection events are considered in our analysis. All of these sensing errors can cause different channel occupancy behavior and lead to different system state transitions.

The transition rate diagram shown in Fig. 1 refers to the situation of the pre-full channel occupancy, i.e., the \( N \) channels are not fully used by both types of calls. Due to the impact of false alarm and misdetection events, the state \((i, j)\) moves to \((i', j+1)\) with transition rate \( [1 - \frac{1}{\mu_1} - \eta(i)p_a \lambda_2] \), where \( \eta(i) = 0 \) if \( i = 0 \) and \( \eta(i) = 1 \) if \( i \neq 0 \), and moves to \((i-1, j)\) with rate \( \mu_1 + p_b \lambda_2 \), where \( \mu_1 \) is the normal transition due to service completion and \( p_b \lambda_2 \) is the additional transition due to class-A misdetection. The state \((i, j)\) moves to \((i, j-1)\) with transition rate \( j \mu_2 + p_a \lambda_1 \), where \( j \mu_2 \) is the normal transition due to service completion\(^2\) and \( p_a \lambda_1 \) is the additional transition due to class-B misdetection. Note that when \( i = 0 \), we have \( p_a = 0 \).

\(^2\)In Fig. 1, type II false alarm may occur but does not affect the state transition, since the ongoing secondary user making type II false alarm can either find another idle channel to continue its service or reconnect back to the system instantly it enters the buffer, according to the proposed model.
In Fig. 2, the state (i, j) moves to (i, j-1) with transition rate \((1-\mu_{p2})j\mu_{2} + \mu_{p2}(j-1)\mu_{1} + r_{2}\), where \((1-\mu_{p2})j\mu_{2}\) is the normal transition without the occurrence of type II false alarm; When a type II false alarm occurs, the corresponding secondary user will leave its current channel and go to the buffer since at this time it cannot find an idle channel (all the channels are occupied), which contributes to \(p_{j2}[j-1]_2\mu_{2} + r_{2}\); and \(p_{b\lambda_{1}}\) is the additional transition due to class-B misdetection. When \(i + j = N\), no initiating ST calls can enter the system, thus, there is only one line between (i, j) and (i, j+1).

In Fig. 3, the system is in the post-full channel occupancy status, no initiating ST call can enter the system. However, it is possible for an ongoing ST call to leave the system due to service completion, type II false alarm, or class-B misdetection, and for a waiting ST call in the buffer to reconnect back due to a completion of a PT or ST call. The state (i, j) means that there are \(i\) PT calls, \(N-i\) ongoing ST calls in the system and \(j-(N-i)\) queued ST calls in the buffer. The state (i, j) moves to (i, j-1) with rate

\[
p_{j2}(N-i-)[(N-i-1)\mu_{2} + (j-N + i+1)r_{2}] + [1-p_{j2}(N-i-)[(N-i)\mu_{2} + (j-N + i)r_{2}] + p_{b\lambda_{1}}(N-i)\lambda_{1},
\]

where the first two terms contribute to the cases without and with the occurrence of type II false alarm, respectively, and the third term contributes to the case when a class-B misdetection occurs.

It is worthy to note that at post-full channel occupancy case, there is at least one ST call waiting in the buffer, which means all the channels have been occupied and thus no type I false alarm and class-A misdetection events can happen. Note also that when \(i = N\), we have \(p_{O} = 0\) and \(p_{b} = 0\); and no initiating PT calls can enter the system.

Let \(\pi(n_{1}, n_{2})\) denote the steady-state probability that the OSS system is in state \((n_{1}, n_{2})\). The steady-state system probability vector, with states ordered lexicographically, can be represented as \(\pi = (\pi_{b}, \pi_{1}, \ldots, \pi_{N})\), where \(\pi_{n} = \pi(n, 0), \pi(n, 1), \ldots, \pi(n, N)\), \(0 \leq n \leq N\). The vector \(\pi\) is the solution of equations \(\pi Q = 0\) and \(\pi e = 1\), where \(e\) and \(0\) are vectors of all ones and zeros, respectively. From the transition rate diagram, the infinitesimal generator \(Q\) of the two dimensional Markov process is obtained as

\[
Q = \begin{bmatrix}
E_{0} & B_{0} & 0 & \cdots & 0 & 0 \\
D_{j} & E_{j} & B_{j} & \cdots & 0 & 0 \\
0 & 0 & \cdots & 0 & D_{N-1} & E_{N-1} \\
0 & 0 & \cdots & 0 & 0 & D_{N} & E_{N}
\end{bmatrix},
\]

where each submatrix has size \((N+1)\) by \((N+1)\) and defined by

\[
B_{i} = (1 - p_{b})\lambda_{i}I_{N+1}, \quad 0 \leq i < N,
\]

\[
D_{j} = \mu_{1}I_{N+1} + p_{a}\lambda_{j}I_{N+1}, \quad 1 \leq j \leq N
\]

\[
E_{j} = A_{j} - \delta(i)D_{j} - \delta(N-i)B_{j}, \quad 0 \leq i \leq N,
\]

where \(I_{n}\) denotes an \(n\)-by-\(n\) identity matrix, and \(A_{j}\) is defined as

\[
I^{(i)} = \begin{bmatrix}
I_{i-1} & 0_{i-1}
\end{bmatrix},
\]

and \(I^{(i)} = I_{i}\). The matrix \(A_{j}\) has the same size as \(E_{j}\). The \((j, k)\) element of \(A_{i}\), denoted by \(A(j, k)\), is derived as

\[
A(j, k) = \begin{cases}
\phi_{j}, & 0 \leq i < N, \quad j = N - i, \quad k = j - 1, \\
\phi_{j}, & 1 \leq i \leq N, \quad j > N - i, \quad k = j - 1, \\
-A_{j}(j-1) + A_{j}(j+1), & 0 \leq i \leq N, \quad 0 \leq j \leq N, \quad k = j,
\end{cases}
\]

where

\[
\phi_{j} = (1 - p_{j2})j\mu_{2} + p_{j2}[(j-1)\mu_{2} + r_{2}] + p_{b}\lambda_{1},
\]

and

\[
\phi_{j} = p_{j2}(N-i)(N-i-1)\mu_{2} + (j-N+i+1)r_{2} + p_{b}\lambda_{1},
\]

Applying the matrix-analytic method developed in [15], the steady state probabilities can be determined as
\[ \pi_n = \pi_0 \prod_{i=0}^{n} \left( B_{i+1} \left(-C_{i} \right)^{-1} \right), \quad 1 \leq n \leq N, \]  
\[ \pi_0 \prod_{n=1}^{\infty} \left( B_{n+1} \left(-C_{n} \right)^{-1} \right) e = 1. \]  

The \( C_i \) can be recursively determined by \( C_N = E_N \) and \( C_i = E_i + B_i \left(-C_{i-1} \right)^{-1} D_{i+1}, \quad 0 \leq i \leq N-1. \) 

There are some special cases for the proposed model.

- **Single Primary System**
  If there are no secondary users in the system, that is, \( n_2 = 0 \), \( \lambda_2 = 0 \), and \( \mu_2 = 0 \), the OSS system reduces to a single primary system. In this case, the performance model simplifies as follows:
  \[ B_i = \lambda_i, \quad 0 \leq i < N; \quad D_i = i \mu_1, \quad 1 \leq i \leq N; \]
  \[ E_i = i \mu_1 - \delta (N-i) \lambda_i, \quad 0 \leq i \leq N; \]
  \[ A_i = 0, \quad 0 \leq i \leq N; \quad C_i = i \mu_1, \quad 0 \leq i \leq N. \]

Substituting the above equations into (2), we obtain
\[ \pi_n = \frac{1}{n!} \left( \frac{\lambda_1}{\mu_1} \right)^n \pi_0, \quad 1 \leq n \leq N, \]
and
\[ \pi_0 = \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{\lambda_1}{\mu_1} \right)^n, \]
which is the well-known Erlang loss model [16].

- **An OSS System with Perfect Sensing: Scenario 1**
  In this scenario, both initial and ongoing secondary users make perfect spectrum detection, i.e., \( p_{D_1} = p_{D_2} = 0 \) and \( p_{A} = p_{B} = 0 \). The resulting OSS system becomes the same as that in [10].

- **An OSS System with Only Initiating Secondary Users Making Sensing Errors: Scenario 2**
  In this scenario, only initiating secondary users make detection errors, i.e., \( p_{D_2} = 0 \) and \( p_{A} = 0 \). The proposed OSS system becomes the same as that in [2].

- **An OSS System with Only Ongoing Secondary Users Making Sensing Errors: Scenario 3**
  In this scenario, only ongoing secondary users make detection errors, i.e., \( p_{D_1} = 0 \) and \( p_{B} = 0 \).

### 4. Performance Metrics

#### 4.1 Blocking Probability of the Primary Traffic

The primary call blocking probability, denoted by \( P_1 \), is defined as the probability that upon an arrival of a PT call in a service area all the channels are occupied by PT calls and no channel is available for a new ST call request. Thus, we have
\[ P_1 = \sum_{n_1=0}^{N} \pi(N,n_1) = \pi_0 \prod_{i=1}^{N} \left( B_{i+1} \left(-C_{i} \right)^{-1} \right) e. \]  

#### 4.2 Blocking Probability of the Secondary Traffic

The secondary call blocking probability, denoted by \( P_2 \), is defined as the probability when all the channels in a service area are occupied by either PT calls and/or ST calls and no channel is available for a new ST call request. Thus, we have
\[ P_2 = \sum_{n_1=0}^{N} \sum_{n_2=0}^{N} \pi(n_1,n_2). \]  

#### 4.3 Total Channel Utilization

The total channel utilization \( \eta \) is defined as the ratio of the mean number of occupied channels to the total number of channels. We find that
\[ \eta = \frac{1}{N} \sum_{n_1=0}^{N} \sum_{n_2=0}^{N} (n_1 + n_2) \pi(n_1,n_2) + \sum_{n_1=1}^{N} \sum_{n_2=0}^{N} N \pi(n_1,n_2). \]  

#### 4.4 Mean Reconnection Probability of Queued ST Calls with False Alarm Ignored

In the proposed model, it is assumed that secondary users know the buffer status. Thus, when there is a queued ST call in the buffer, the type I false alarm and class-A misdetection events are impossible to happen. However, the type II false alarm and class-B misdetection events may occur. An ongoing ST call may fail to detect an arriving primary call at its channel, and thus leads to a collision with the primary call, causing the dropping of both calls. An ongoing ST call may also incorrectly determine that a primary call is arriving at its channel, so it leaves its channel and enters the end of the queue in the buffer. Then, the HOL ST call immediately reconnects back to the released channel. As far as the “mean” reconnection event is concerned, it does not distinguish the specific ST calls. In this section, we prefer to study the mean reconnection probability of the queued ST calls by not considering the impact of false alarm event.

Let \( \gamma \) denote the mean reconnection probability of the queued ST calls in the buffer, which involves all of the possible queued ST calls with \( n_1 \) possible primary calls in the system, \( 1 \leq n_1 \leq N \). Let \( r \) denote the maximum queuing time that can be tolerated by an ST call in the buffer. The maximum waiting time of an ST call in the buffer is assumed to be statistically the same as the residence time of the ST call. Hence, \( r \) is exponentially distributed with mean \( 1/r \).

To capture the queueing behavior of the queued ST calls, we introduce a 3-dimensional Markov process \( (Z(t), X(t), J(t)) \) under the condition of post-full channel occupancy, where \( Z(t), X(t), \) and \( J(t) \) represent the number of primary calls, ongoing secondary calls, and queued secondary calls in the system at time \( t \). The state space of the Markov process is
\[ S^* = \{(n_1, n_2, j) \mid n_1 + n_2 = N, 0 \leq j \leq N \}. \]

Actually, the above 3-dimensional Markov process \( (Z(t), X(t), J(t)) \) is equivalent to the previous 2-dimensional Markov process \( (X_i(t), X_{i+1}(t)) \) in Section 3 under the post-full condition, where \( Z_i(t), X_i(t), \) and \( J(t) \) can be determined by \( X_i(t) \) and \( X_{i+1}(t) \), and vice versa. Suppose that an ST call arriving at the buffer finds that there are \( J \) ST calls in the buffer, \( 0 \leq J \leq N-1 \). The system state can be represented as \( (n_1, N-n_1, j+1) \) with \( 0 \leq j \leq N-1 \). Let \( \gamma(j) \) denote the probability that an ST call arriving at the buffer eventually reconnects to the system before its maximum queuing time expires, given that the ST call comes to find that there are \( J \) ST calls in the buffer \( (0 \leq j \leq N-1) \). The mean reconnection probability \( \gamma \) can be expressed as
\[
\gamma = \frac{\sum_{n=1}^{N-1} \sum_{j=0}^{n-1} \pi(n, N-n_j + j + 1) \gamma(f)}{\sum_{n=1}^{N-1} \sum_{j=0}^{n-1} \pi(n, N-n_j + j + 1)}.
\]

(8)

To derive the conditional probability \( \gamma(j) \), we analyze the following possible events and their effects on system state transition, given the current system state \((n_1, N-n_1, j+1)\):

- If an ongoing ST call correctly detects an arrival of a PT call at its channel, it releases its channel for the PT call and enters the buffer. The system state changes to \((n_1+1, N-n_1-1, j+2)\).
- If an ongoing ST call makes a type II false alarm detection, it releases its channel and enters the buffer, and the HOL ST call in the buffer immediately reconnects to the channel. As a result, the system state does not change.
- If an ongoing ST call makes a class-B misdetection, it reconnects to the channel. This leads to a new system state \((n_1, N-n_1, j)\).
- If the maximum waiting time of a queued ST call expires, it is dropped from the system. The remaining queued ST calls that were behind it advance by one position in the buffer. The system state changes to \((n_1, N-n_1, j)\).
- If a PT call completes its service, it leaves the system, and the HOL ST call in the buffer immediately reconnects to the channel. This leads to a new system state \((n_1-1, N-n_1+1, j)\).
- If an ST call completes its service, it leaves the system, and the HOL ST call in the buffer immediately reconnects to the channel. This leads to a new system state \((n_1, N-n_1, j)\).

Let \( \phi_j \) denote the time interval, in steady-state, between a transition to a state \((n_1, N-n_1, j+1)\) until a transition to a new state \((n_1', N-n_1', j)\), due to the service completion of a PT/ST call, the class-B misdetection of an ongoing ST call, or the dropping of a queued ST call from the system. If a PT call leaves due to service completion, then \(n_1' = n_1-1\); otherwise, \(n_1' = n_1\). Hence, \( \phi_j \) is exponentially distributed with parameter \( g_j \), \( 0 \leq j \leq N-1 \), given by

\[
g_j = n_j \mu + (N-n_j) \mu_2 + p_b \lambda_2 + j \tau_2. \]

(9)

Let \( f_{\phi_j} \) denote the probability density function (pdf) of \( \phi_j \) and let \( f_{\phi_j}^* \) denote the Laplace transform\(^3\) of \( f_{\phi_j} \). By the independence assumption of the random variables \( \phi_j \), we can determine \( \gamma(j) \) as

\[
\gamma(j) = \Pr(\tau > \phi_0 + \phi_1 + \cdots + \phi_j) = \prod_{i=0}^{j} f_{\phi_i}^*(\tau_i)
\]

(10)

\[
= \frac{n_j \mu + (N-n_j) \mu_2 + p_b \lambda_2}{n_1 \mu + (N-n_1) \mu_2 + p_b \lambda_2 + (j+1) \tau_2},
\]

where the last equation follows from the fact that

\[
f_{\phi_i}^*(\tau_i) = \frac{n_i \mu + (N-n_i) \mu_2 + p_b \lambda_2 + i \tau_2}{n_i \mu + (N-n_i) \mu_2 + p_b \lambda_2 + (i+1) \tau_2}.
\]

The mean reconnection probability \( \gamma \) can then be calculated by substituting (10) into (8).

5. Numerical Results

In this section, we present the numerical results for the obtained performance metrics in the following configuration: \( N = 16, \mu_1 = 15, \mu_2 = 15, r_2 = 5, \lambda_2 = 60 \), the primary traffic intensity \( \rho_1 \) (defined as \( \lambda_2/\mu_2 \)) changes from 1 to 7. We set the parameters \( p_{b1}, p_{b2}, p_a \) and \( p_b \) separately in each figure, and study the impact of unreliable sensing on system performance and compare them with the system performance under different scenarios such as a single primary system, Scenario 1, Scenario 2, and Scenario 3. We omit the performance curves with respect to the change of \( p_{b2} \) (defined as \( \lambda_2/\mu_2 \)) due to space limitation. Note that all parameters are given in dimensionless units, which can be mapped to specific units of measurement. Note also that there are no specific requirements for the above parameter configuration, i.e., any reasonable parameter configuration can be used for the numerical computation.

Fig. 4 shows how the primary call blocking probability \( P_1 \) changes with various parameters. We observe that \( P_1 \) increases with the increase of \( \rho_1 \) and does not change with the change of false alarm probabilities. On the other hand, we observe that \( P_1 \) decreases a little with the increase of the misdetection probability \( p_a, p_b \), or both. The reason is that the misdetection event causes the release of a working channel, leading to an additional opportunity for new incoming PT call requests. In addition, we also observe that \( P_1 \) is the same in Scenario 1 and in a single primary system, which indicates that in perfect spectrum sensing condition, the introduction of secondary system does not affect the performance of the primary system.

Fig. 5 shows how the secondary call blocking probability \( P_2 \) changes with various parameters. We observe that \( P_2 \) increases with the increase of \( \rho_1 \). This is because an ST call has to contend with more PT calls for the fixed number of channels. We also observe the impact of unreliable sensing on \( P_2 \). When \( p_{b1}, p_{b2} \), or \( p_b \) increases, \( P_2 \) will decrease. When a false alarm event occurs, the channel remains idle or is forced to be idle, potentially to be used by other ST call requests. On the other hand, an occurrence of a misdetection event directly results in a release of a channel that was being used by either a primary user or a secondary user, leading to an additional opportunity for new incoming PT call requests. However, a misdetection event clearly degrades the performance of the OSS system as seen by primary users. In addition, by comparing Figs. 4 and 5, we observe that the ST call blocking probability is higher than that of PT calls under the same parameter settings, as should be expected.

In Fig. 6, we observe that the channel utilization of the OSS system \( \eta \) increases with the increase of \( \rho_1 \) or \( \rho_2 \), and decreases as the probability of spectrum sensing error increases. For example, the channel utilization in Scenario 2 or 3 decreases as compared to that in Scenario 1 (perfect sensing), and increases as compared to the case that \( p_{b1} = p_{b2} = p_a = p_b = 0.2 \). A type I false alarm event wastes an idle channel, while a type II false alarm or any misdetection event not only wastes an idle channel but also causes an active channel to

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become an idle channel. Particularly, a misdetection event also makes a primary call drop from the system, causing a harmful interference to the primary user. Thus, a misdetection degrades system performance more severely than a false alarm event. It is also observed that the channel utilization of the OSS system is much higher than that of the single primary system, even in the condition of unreliable sensing, with appropriate sensing errors. Only when both types of users make very large sensing errors, e.g., \( p_f = p_a = 0.2 \), the channel utilization of the OSS system may, under relevantly large primary traffic intensity condition, become lower than the single primary system.

The initiating secondary users sense channels that are unused by the primary users and utilize the idle channels. An ongoing secondary user also detects when a primary user accesses its channel and then either moves to another idle channel or moves to a buffer if no idle channel is available. Unreliable spectrum sensing was modeled by false alarm and misdetection events for both initiating and ongoing secondary users. We solved the steady-state probability vector of the system and derive a set of performance metrics of interest. Numerical results are presented to highlight the analysis. The proposed modeling method can be used to evaluate the performance of future opportunistic spectrum sharing networks.

Fig. 7 shows the impact of various parameters on the mean reconnection probability with false alarm ignored, \( \gamma \). We observe that \( \gamma \) decreases as \( \rho_1 \) increases, and increases as the mean value, \( E[\tau] \), of the maximum ST call queueing time \( \tau \) (see Section 4.4), is increased. The reason is as follows: a higher volume of PT calls results in a smaller chance that a queued ST call reconnects to the system, while a longer maximum queueing time leads to a higher chance of reconnection. As should be expected, we observe that \( \gamma \) is basically the same in Scenario 1 and Scenario 2 and does not change with \( \rho_2 \). It can also be seen that \( \gamma \) increases with the increase of \( p_b \). As \( p_b \) increases, more channels being used by ST calls tend to be released, leading to a higher chance of reconnection.

6. Conclusions

We built a Markov model to analyze the types of unreliable spectrum sensing and their impact on the system performance in an opportunistic spectrum sharing wireless system. The system consists of the primary users and the secondary users. The initiating secondary users sense channels that are unused by the primary users and utilize the idle channels. An ongoing secondary user also detects when a primary user accesses its channel and then either moves to another idle channel or moves to a buffer if no idle channel is available. Unreliable spectrum sensing was modeled by false alarm and misdetection events for both initiating and ongoing secondary users. We solved the steady-state probability vector of the system and derive a set of performance metrics of interest. Numerical results are presented to highlight the analysis. The proposed modeling method can be used to evaluate the performance of future opportunistic spectrum sharing networks.
References


